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TRAJECTORIES OF AN UNGUIDED PROBE WITH A DAMPING CABLE

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The relative orbital motion of two point masses connected by a flexible nonexpadable weightless cord with shock contacts is considered. This system models, for example, the dynamics of a satellite and a probe connected by a cable. The orbit of the center of mass is assumed to be circular. The motion of the probe relative to the center of mass can be presented as a series of portions of free motion between successive contacts, i.e., as the probe recedes from the satellite to a distance equal to the cable length. At the time of establishing contact, there is shock interaction, which can be of various nature – perfectly elastic, perfectly inelastic, and with partial absorption of energy.

We consider a portion of free motion, i.e., while the cable connecting the probe and the satellite is not stretched. We will assume the orbit of the center of mass of the system, point O, to be circular and motion of the probe to occur in the plane of this orbit. Let a system of coordinates O, $\bar{\eta}$, $\bar{\xi}$, $\bar{\zeta}$ whose axes are directed all the time along the radius-vector of the orbit of the satellite ($\bar{\eta}$), the transversal ($\bar{\xi}$), and the binormal to the plane of the orbit ($\bar{\zeta}$) be related to the center of mass of the satellite–probe system.

We introduce dimensionless coordinates and dimensionless velocity of the center of mass of the probe

$$(\xi, \eta) = \vec{\rho} = \frac{\vec{r}}{lm_0/(m + m_0)}, \quad (\xi', \eta') = \vec{v} = \frac{\vec{V}}{\omega_0 l}.$$

For a circular orbit of the satellite in the case of plane motion of the probe, the current coordinates η and ξ of the probe satisfy the equations [1]

$$\eta'' - 2\xi' - 3\eta = 0, \quad \xi'' + 2\eta' = 0. \tag{1}$$

The primes in (1) denote derivatives with respect to the dimensionless time $\tau = \omega_0 t$, where ω_0 is the angular velocity of motion of the center of mass of the system around its orbit; here the dimensionless orbital period is 2π. The integral of energy in the form of

$$\eta'^2 + \xi'^2 - 3\eta^2 = h = \text{const}. \tag{2}$$

follows from Eqs. (1). Equations (1) describe the motion of the probe only until the cable is stretched. They can easily be integrated

$$\eta = 2c_1 + c_2 \sin \tau + c_3 \cos \tau, \quad \xi = c_4 - 3c_1\tau + 2c_2 \cos \tau - 2c_3 \sin \tau. \tag{3}$$

Here the arbitrary constants c_1, c_2, c_3, and c_4 are expressed in terms of the initial (at $\tau = 0$) data as follows:

$$c_1 = 2\eta_0 + \xi_0', \quad c_2 = \eta_0',$$

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$$c_3 = -3\eta_0 - 2\xi_0', \quad c_4 = \xi_0 - 2\eta_0'. \tag{4}$$

All the statements above on the motion of the probe refer to its free motion. In the general case, the probe inevitably moves away from the center of mass of the system (the probability of reaching the periodic trajectory is low) and at a certain instant of time it will move away to the full length of the cable. Further behavior of the probe greatly depends on the elastic properties of the cable, i.e., by formulation, on the character of shock when the system establishes contact.

We consider the behavior of the system at the time of shock. We know the solution (3) of the system of equations (1) that describes the free motion of the probe until the cable is stretched. Using this solution, from the condition $\eta^2 + \xi^2 = 1$ we can find the instant of time τ_* at which the probe moves away from the center of mass of the system to a distance l equal to the cable length. In this case, τ_* satisfies the equation

$$9c_1^2\tau_*^2 + 3c_2^2\cos^2\tau_* + 3c_3^2\sin^2\tau_* - 6c_2c_3\cos\tau_*\sin\tau_* - 12c_1c_2\tau_*\cos\tau_* +$$

$$+ 12c_1c_2\tau_*\sin\tau_* + 4(c_1c_3 + c_2c_4)\cos\tau_* + 4(c_1c_2 + c_3c_4)\sin\tau_* - 6c_1c_4\tau_* +$$

$$+ 4c_1^2 + c_2^2 + c_3^2 + c_4^2 - 1 = 0. \tag{5}$$

Hence, determining numerically $\tau_n = \tau_*$ for the nth portion of the trajectory, we can write the corresponding values of the coordinates and velocities

$$\eta_n = 2c_1 + c_2\sin\tau_* + c_3\cos\tau_n, \quad \xi_n = c_4 - 3c_1\tau_n + 2c_2\cos\tau_n - 2c_3\sin\tau_n,$$

$$\eta_n' = c_2\cos\tau_n - c_3\sin\tau_n, \quad \xi_n' = -3c_1 - 2c_2\sin\tau_n - 2c_3\cos\tau_n.$$

At the instant of time τ_n, depending on the assumed character of shock the velocity component v_n^r, directed along the cable away from the center of mass of the system, reverses its direction and varies in magnitude. The transverse component of velocity v_n^τ, directed along the normal to the stretched cable, still persists. Now, we can write the initial data (coordinates and velocities) at the beginning of the next $(n+1)$ portion of free motion

$$\xi_{n+1} = \xi_n, \quad \eta_{n+1} = \eta_n, \quad v_{n+1}^\tau = v_n^\tau, \quad v_{n+1}^r = -kv_n^r. \tag{6}$$

Here k is the restoration factor, a quantity which is determined by the character of the shock: $k = 1$, a perfectly elastic shock, $k = 0$, a perfectly inelastic shock, and $0 < k < 1$, a shock with partial absorption of energy. The character of the shock will depend on the properties of the cable and its ability to damp.

Having written the equation of velocity variation with shock, after simple transformations we obtain the equations of velocity variation in shock interaction

$$\begin{aligned} \eta_{n+1}' &= \eta_n' - (1+k)\,\eta_n\,(\xi_n'\xi_n + \eta_n'\eta_n), \\ \xi_{n+1}' &= \xi_n' - (1+k)\,\xi_n\,(\xi_n'\xi_n + \eta_n'\eta_n). \end{aligned} \tag{7}$$

Equations (7) for the case of a perfectly elastic shock are given in [1].

As a result we obtain the constructed algorithm $\{\vec{\rho}_{n+1}, \vec{v}_{n+1}\} = \Phi\{\vec{\rho}_n, \vec{v}_n\}$ of calculation of successive shocks:

1) knowing the initial coordinates and velocities for the nth portion of the trajectory, we find the time of shock τ_n by formula (5);

2) using formulas (6), we find the initial data for the next $(n+1)$ portion of the trajectory $\{\vec{\rho}_{n+1}, \vec{v}_{n+1}\}$, etc.

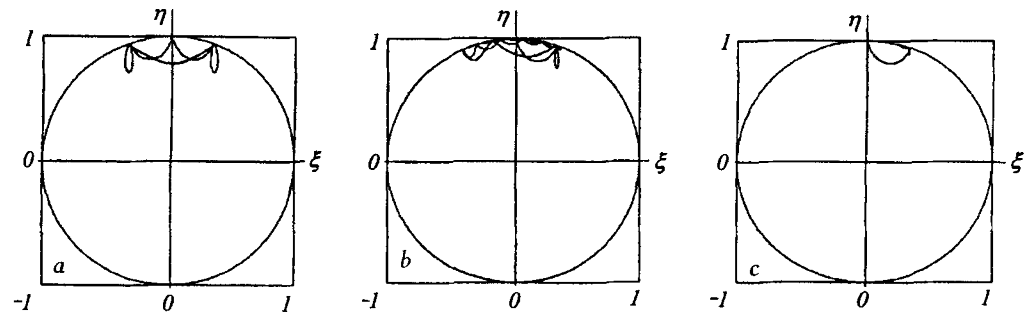

Fig. 1. Trajectory of the probe at $h = -1.5$, $k = 1$ (a), $k = 0.9$ (b), and $k = 0.5$ (c).

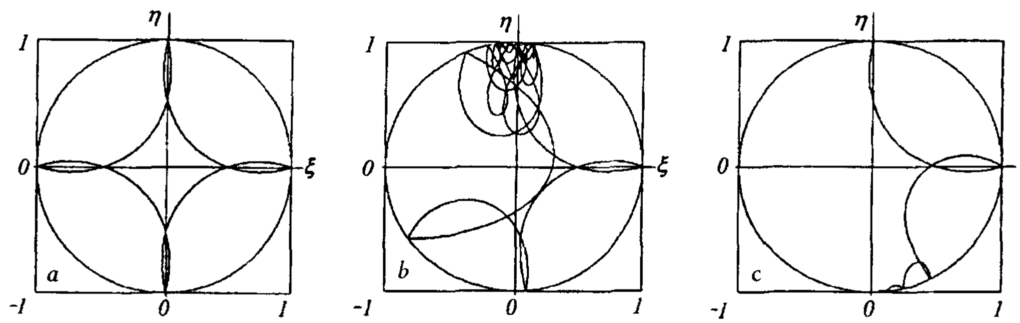

Fig. 2. Trajectory of the probe at $h = 2.5$, $k = 1$ (a), $k = 0.9$ (b), and $k = 0.5$ (c),

It is characteristic of a perfectly elastic shock ($k = 1$) that in motion, the integral of energy (2) remains constant not only on the portions of free motion, but also when contacts are established. In shock, the transverse component of velocity varies neither in magnitude nor in direction, whereas the radial component, remaining constant in magnitude, reverses its direction. Figures 1a and 2a present, for comparison with the case $k \neq 1$, some possible periodic trajectories for $h = -1.5$ and $h = 2.5$.

With a perfectly inelastic shock ($k = 0$) there is only one portion of free motion, since at the time of the very first contact the radial component of velocity, by virtue of shock inelasticity, will be damped completely and the system will move as a solid unit. The model of a "rigid" orbital cable system (OCS) is appropriate for describing the motion of this cable system.

In the case of a shock with partial absorption of energy ($0 < k < 1$), the constant energy will remain invariable only on the portions of free motion. At the time of shock it will begin to change, constantly decreasing due to the damping of the radial component of velocity with a succession of shocks. The degree of velocity damping depends on the ability of the cable to damp, i.e., on the material the cable is made of and its structure (number of threads and layers and method of thread laying). At a certain stage of motion the radial component of velocity will become negligibly small and then the model of a "rigid" OCS can be used for describing further motion (similar to a perfectly inelastic shock). But in this case transition to a "rigid" OCS is accomplished with the very first shock, while with partial absorption of energy – only after a series of these shocks. The transition time depends on the following conditions: the characteristics of the cable (restoration factor), the constant energy, and the initial values of motion parameters. In the case of identical constant energy and characteristics of the cable, the transition time will be larger if the velocity of motion at the initial instant is directed along the radius-vector.

Figure 1b and c presents the trajectories of motion at different restoration factors, $k = 0.9$ and $k = 0.5$ for $h = -1.5$ and the initial conditions $(\xi_0, \eta_0) = (0, 1)$, $(\xi_0', \eta_0') = (0.32, -1.18)$, and Fig. 2b and c shows the trajectories of motion at $h = 2.5$, $k = 0.9$, $k = 0.5$ and the initial conditions $(\xi_0, \eta_0) = (0, 1)$, $(\xi_0', \eta_0') = (-0.49, -2.29)$. The time of transition to a "rigid" OCS will be maximum at some nonmaximum value of the radial component of velocity. We note that the dependences of the transition time on the restoration factor and the direction of velocity at the initial instant significantly differ at negative and positive h.

NOTATION

$O\bar{\eta}\bar{\xi}\bar{\zeta}$, orbital system of coordinates; $\vec{r}$, radius-vector of the center of mass of the probe relative to the center of mass of the system; $\vec{V}$, velocity vector of the center of mass of the probe; η, ξ, dimensionless coordinates of the center of mass of the probe; $\vec{v}$, dimensionless velocity vector of the center of mass of the probe; τ, dimensionless time; m, mass of the probe; m_0, mass of the satellite; l, length of the cable; h, value of the integral of energy; k, restoration factor.

REFERENCE

1. V. V. Beletsky and D. V. Pankova, *Regular and Chaotic Dynamics*, No. 1, 87-103 [in Russian], Moscow (1996).